Theorem

$\lim_{x \to a} f(x) = L$ if and only if

$$\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$$

Ex: If

$$f(x) = \begin{cases} x^2+1 & \text{if } x < 1 \\ (x-2)^2 & \text{if } x \geq 1 \end{cases}$$

does $\lim_{x \to 1} f(x)$ exist?

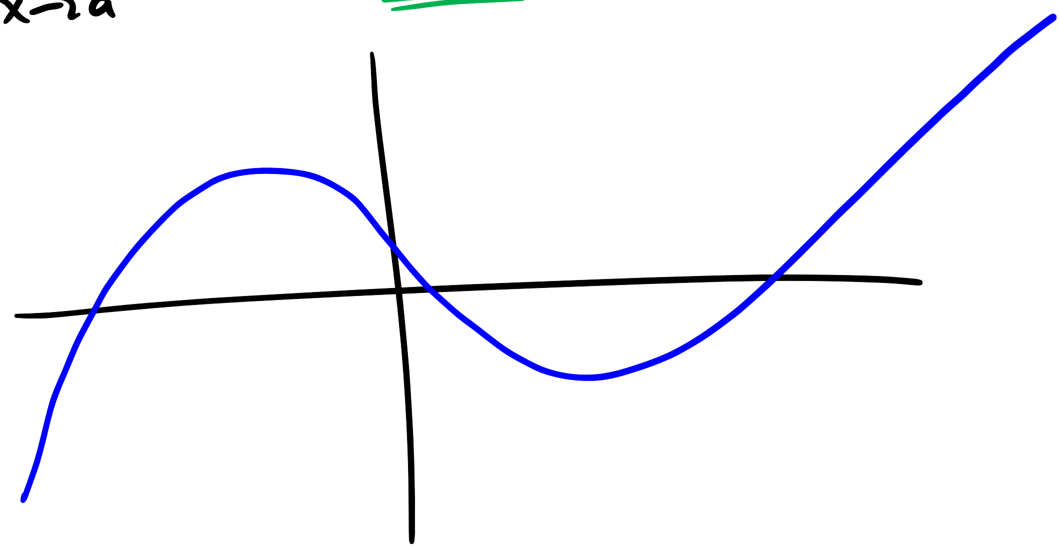
$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^2+1) = 1^2+1 = 2$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x-2)^2 = (1-2)^2 = 1$$

So, $\lim_{x \to 1} f(x)$ DNE (left/right limits are different)

Continuity

Def: A function f is <u>continuous at $x=a$</u> if $\lim_{x\to a} f(x) = f(a)$.



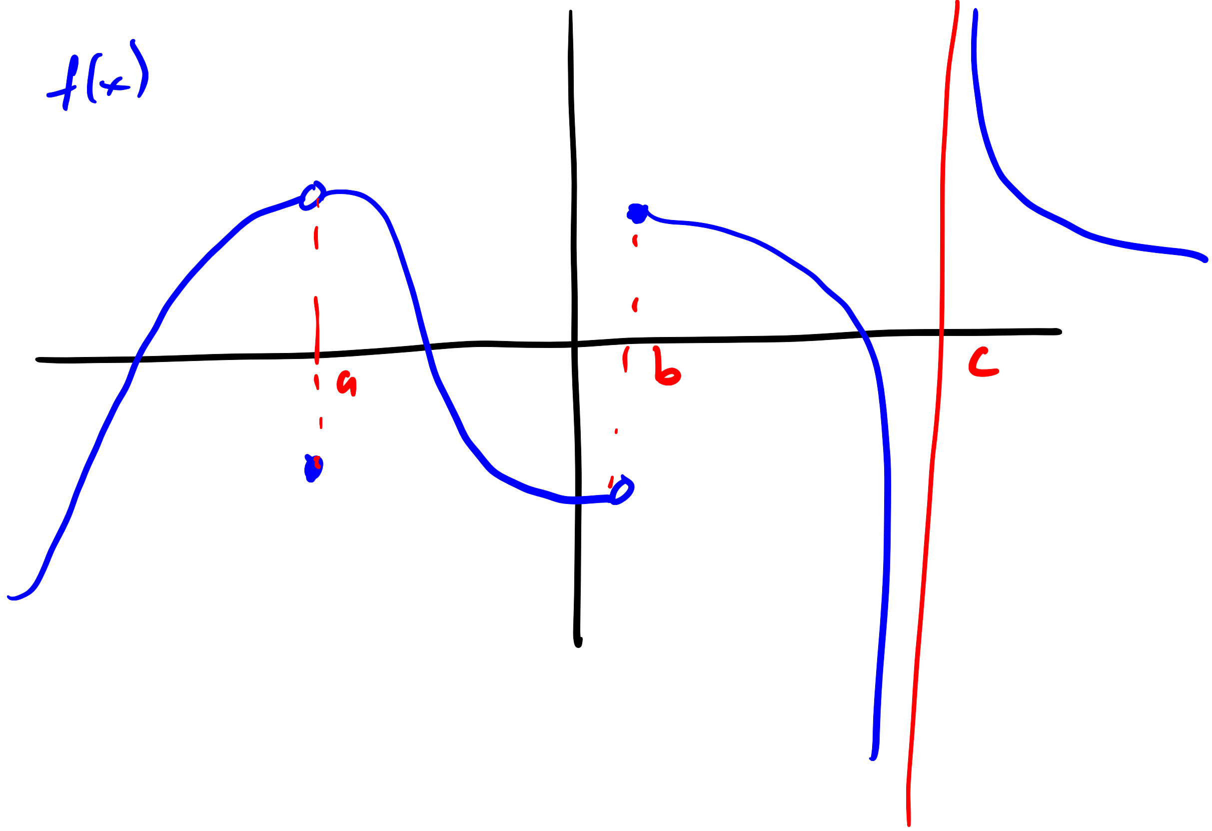
This definition implies 3 things:

① $f(a)$ is defined (a is in the domain of f)

② $\lim_{x\to a} f(x)$ exists

③ $\lim_{x\to a} f(x) = f(a)$

If f is not continuous at $x=a$, then we say f is <u>discontinuous at $x=a$</u> or <u>f has a discontinuity at $x=a$.</u>



3 discontinuities:

① at $x=a$, f has a removable discontinuity

② at $x=b$, f has a jump discontinuity

③ at $x=c$, f has an infinite discontinuity

Ex: Where is $f(x)=\dfrac{x^3-x^2-2x}{x-2}$ discontinuous? What type of discontinuity is it?

Sol:

f is discontinuous @ $x=2$

(f is continuous on $(-\infty, 2) \cup (2, \infty)$)

$$\frac{x^3 - x^2 - 2x}{x-2} = \frac{x(x^2 - x - 2)}{x-2} = \frac{x\cancel{(x-2)}(x+1)}{\cancel{x-2}}$$

$$= x(x+1)$$

f with the discontinuity removed

The discontinuity is removable

<u>Def</u>: The function f is <u>continuous on</u> the interval I if it is continuous at every point in the interval.

<u>Theorem</u>: If f & g are continuous at $x=a$ and c is a constant, then the following are continuous at $x=a$:

① $f \pm g$ ② cf ③ $f \cdot g$

④ $\frac{f}{g}$ if $g(a) \neq 0$.

Which functions are continuous?

Since $\lim_{x \to a} x = a$ for every number a, then $f(x) = x$ is continuous. Likewise since $\lim_{x \to a} c = c$ for every number a, then $f(x) = c$ is continuous. These are continuous on $(-\infty, \infty)$.

- Polynomials are continuous on $(-\infty, \infty)$.
- Rational functions are continuous on their domain.
- Algebraic functions are continuous on their domain.
- Trig, inverse trig, exponential, logarithmic